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Decreasing Inventory Levels Fluctuations by Moving Horizon Control Method and Move Suppression in the Demand Network

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Abstract— The significance of the basic idea implicit in the moving horizon control (MHC) has been recognized a long time ago in the operations management literature as a tractable scheme for solving stochastic multi period optimization problems, such as production planning and supply chain management, under the term moving horizon. In this paper, a moving horizon controller with move suppression term used for inventory management of the demand network (supply chain).

Index Terms— moving horizon control, production planning, supply chain management, move suppression term, inventory management ,demand network.

1 Introduction

Key elements to an efficient supply chain are accurate pinpointing of process flows and timing of supply needs at each entity, both of which enable entities to request items as they are needed, thereby reducing safety stock levels to free space and capital. The operational planning and direct control of the network can in principle be addressed by a variety of methods, including deterministic analytical models and stochastic analytical models, and simulation models, coupled with the desired optimization objectives and network performance measures [1].

The significance of the basic idea implicit in the moving horizon control (MHC) or MHC has been recognized a long time ago in the operations management literature as a tractable scheme for solving stochastic multi period optimization problems, such as production planning and supply chain management, under the term moving horizon [2]. In a recent paper [3], a MHC strategy was employed for the optimization of production/distribution systems, including a simplified scheduling model for the manufacturing function. The suggested control strategy considers only deterministic type of demand, which reduces the need for an inventory control mechanism [4,5].

For the purposes of our study and the time scales of interest, a discrete time difference model is developed [6]. The model is applicable to multi echelon supply chain networks of arbitrary structure. To treat process uncertainty within the deterministic supply chain network model, a MHC approach is suggested [7,8].

Typically, MHC is implemented in a centralized fashion [9]. The algorithm uses a moving horizon, to allow the incorporation of past and present control actions to future predictions [10,11,12,13].

In this paper, a moving horizon controller with move suppression term used for inventory management of the demand network.

2 Modelling and Control

In this work, a discrete time difference model is developed[4]. The model is applicable to multi echelon supply chain networks of arbitrary structure, that *DP* denote the set of desired products in the supply Chain and these can be manufactured at plants, *P*, by utilizing various resources, *RS*. The manufacturing function considers independent production lines for the distributed products. The products are subsequently transported to and stored at warehouses, *W*. Products from warehouses are transported upon customer demand, either to distribution centers, *D*, or directly to retailers, *R*. Retailers receive time varying orders from different customers for different products. Satisfaction of customer demand is the primary target in the supply chain management mechanism. Unsatisfied demand is recorded as backorders for the next time period. A discrete time difference model is used for description of the supply chain network dynamics. It is assumed that decisions are taken within equally spaced time periods (e.g. hours, days, or weeks). The duration of the base time period depends on the dynamic characteristics of the network. As a result, dynamics of higher frequency than that of the selected time scale are considered negligible and completely attenuated by the network [4,14]. Plants *P*, warehouses *W*, distribution centers *D*, and retailers *R* constitute the nodes of the system. For each node, *k*, there is a set of upstream nodes and a set of

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downstream nodes, indexed by (k',k''). Upstream nodes can supply node *k* and downstream nodes can be supplied by *k*. All valid (k',k) and/or (k,k'') pairs constitute permissible routes within the network. All variables in the supply chain network (e.g. inventory, transportation loads) valid for bulk commodities and products. For unit products, continuous variables can still be utilized, with the addition of a post processing rounding step to identify neighbouring integer solutions. This approach, though clearly not formally optimal, may be necessary to retain computational tractability in systems of industrial relevance.

A product balance around any network node involves the inventory level in the node at time instances *t* and *t* − 1, as well as the total inflow of products from upstream nodes and total outflow to downstream nodes. The following balance equation is valid for nodes that are either warehouses or distribution centers:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - \sum_{k''} x_{i,k,k''}(t), \quad \forall k \in \{W, D\}, \quad t \in T, \quad i \in DP \tag{1}$$

where $y_{i,k}$ is the inventory of product *i* stored in node *k*; $x_{i,k',k}$ denotes the amount of the *i*-th product transported through route (k',k); $L_{k',k}$ denotes the transportation lag (delay time) for route (k',k), i.e. the required time periods for the transfer of material from the supplying node to the current node. The transportation lag is assumed to be an integer multiple of the base time period.

For retailer nodes, the inventory balance is slightly modified to account for the actual delivery of the *i*-th product attained, denoted by $d_{i,k}(t)$.

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - d_{i,k}(t), \quad \forall k \in \{R\}, \quad t \in T, \quad i \in DP. \tag{2}$$

The amount of unsatisfied demand is recorded as back-orders for each product and time period. Hence, the balance equation for back orders takes the following form:

$$BO_{i,k}(t) = BO_{i,k}(t-1) + R_{i,k}(t) - d_{i,k}(t) - LO_{i,k}(t), \quad \forall k \in \{R\}, \quad t \in T, \quad i \in DP. \tag{3}$$

where $R_{i,k}$ denotes the demand for the *i*-th product at the *k*-th retailer node and time period *t*. $LO_{i,k}$ denotes the amount of cancelled back orders (lost orders) because the network failed to satisfy them within a reasonable time limit. Lost orders are usually expressed as a percentage of unsatisfied demand at time *t*. Note that the model does not require a separate balance for customer orders at nodes other than the final retailer nodes [4,15].

MHC originated in the late seventies and has developed considerably since then. The term MHC does not designate a specific control strategy but rather an ample range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function. The ideas, appearing in greater or lesser degree in the predictive control family, are basically the explicit use of a model to predict the process output at future time instants (horizon), the calculation of a control sequence minimizing an objective function and the use of a moving strategy, so that at each instant the horizon is displaced towards the future, which involves the application of the first control signal of the sequence calculated at each step. The success of MHC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite horizon strategy allows the explicit handling of process and operational constraints by the MHC. The control system aims at operating the supply chain at the optimal point despite the influence of demand changes [12,13]. The control system is required to possess built in capabilities to recognize the optimal operating policy through meaningful and descriptive cost performance indicators and mechanisms to successfully alleviate the detrimental effects of demand uncertainty and variability. The main objectives of the control strategy for the supply chain network can be summarized as follows: (i) maximize customer satisfaction, and (ii) minimize supply chain operating costs.

The first target can be attained by the minimization of back orders (i.e. unsatisfied demand) over a time period because unsatisfied demand would have a strong impact on company reputation and subsequently on future demand and total revenues. The second goal can be achieved by the minimization of the operating costs that include transportation and inventory costs that can be further divided into storage costs and inventory assets in the supply chain network. Based on the fact that past and present control actions affect the future response of the system, a moving time horizon is selected. Over the specified time horizon the future behavior of the supply chain is predicted using the described difference model (Eqs. (1)–(3)). In this model, the state variables are the product inventory levels at the storage nodes, *y*, and the back orders, *BO*, at the order receiving nodes. The manipulated (control or decision) variables are the product quantities transferred through the network's permissible routes, *x*, and the delivered amounts to customers, *d*. Finally, the product back orders, *BO*, are also matched to the output variables. The inventory target levels (e.g. inventory set-points) are time invariant parameters. The control actions that minimize a performance index associated with the outlined control objectives are then calculated over the moving time horizon. At each time period the first control action in the calculated sequence is implemented. The effect of unmeasured demand disturbances and model mismatch is computed through comparison of the actual

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current demand value and the prediction from a stochastic disturbance model for the demand variability. The difference that describes the overall demand uncertainty and system variability is fed back into the MHC scheme at each time period facilitating the corrective action that is required.

The centralized mathematical formulation of the performance index considering simultaneously back orders, transportation and inventory costs takes the following form[4]:

$$
\begin{aligned}
J_{total} = & \\
& \sum_{t}^{t+P} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{y,i,k} (y_{i,k}(t) - y_{s,i,k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\
& + \sum_{t}^{t+P} \sum_{k\in\{R\}} \sum_{i\in DP} \left\{ w_{BO,i,k} (BO_{i,k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}
\end{aligned}
\tag{4}
$$

The performance index, J, in compliance with the outlined control objectives consists of four quadratic terms. Two terms account for inventory and transportation costs throughout the supply chain over the specified prediction and control horizons (P , M). A term penalizes back orders for all products at all order receiving nodes (e.g. retailers) over the moving horizon P. Also a term penalizes deviations for the decision variables (i.e. transported product quantities) from the corresponding value in the previous time period over the control horizon M. The term is equivalent to a penalty on the rate of change in the manipulated variables and can be viewed as a move suppression term for the control system. Such a policy tends to eliminate abrupt and aggressive control actions and subsequently, safeguard the network from saturation and undesired excessive variability induced by sudden demand changes. In addition, transportation activities are usually preferred to resume a somewhat constant level rather than fluctuate from one time period to another.

However, the move suppression term would definitely affect control performance leading to a more sluggish dynamic response. The weighting factors, $w_{y,i,k}$, reflect the inventory storage costs and inventory assets per unit product, $w_{x,i,k',k}$, account for the transportation cost per unit product for route (k',k).Weights $w_{BO,i,k}$ correspond to the penalty imposed on unsatisfied demand and are estimated based on the impact service level has on the company reputation and future demand. Weights $w_{\Delta x,i,k',k}$, are associated with the penalty on the rate of change for the transferred amount of the i-th product through route (k',k). Even though, factors $w_{y,i,k}$, $w_{x,i,k',k}$ and $w_{BO,i,k}$ are cost related that can be estimated with a relatively good accuracy, factors $w_{\Delta x,i,k',k}$ are judged and selected mainly on grounds of desirable achieved performance.

The weighting factors in cost function also reflect the relative importance between the controlled (back orders and inventories) and manipulated (transported products) variables. Note that the performance index of cost function reflects the implicit assumption of a constant profit margin for each product or product family. As a result, production costs and revenues are not included in the index.

In this centralized implementation, MHC will optimized for whole policy and then will sent downstream optimal inputs to upstream joint nodes to those nodes which it is coupled, as measurable disturbances.

Each node completely by a centralized MHC optimizes for whole policy. At each time period, the first control action in the calculated sequence is implemented until MHC process complete.

3 SIMULATIONS

A four echelon supply chain system is used in the simulated examples. The supply chain network consists of one product, two production nodes, two warehouses, four distribution centers, and four retailer nodes.

All possible connections between immediately successive echelons are permitted. One product is being distributed through the network. Inventory setpoints, maximum storage capacities at every node, and transportation cost data for each supplying route are reported in Table 1.

A prediction horizon of 20 time periods and a control horizon of 10 time periods were selected and was considered $LO = 3$ for every times. So each delay was replaced by its 4th order Pade approximation.

Table 1. Supply chain data

Echelon	W	D	R
Max inventory level	1000	700	300
Inventory setpoint	500	300	180
Transportation cost	P to W $\begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$	W to D $\begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.2 \\ 0.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^T$	D to R $\begin{bmatrix} 0.5 & 0.2 & 0.5 & 0.5 \\ 0.2 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.2 \\ 0.5 & 0.5 & 0.2 & 0.5 \end{bmatrix}$
Inventory weights	1	1	1
Back-order weights		-	1
Delivery weights	-	-	-0.5
Move suppression weights term	3	**3**	3
Initial condition	500	300	180
Delays	5	3	2

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The simulated scenarios lasted for 50 time periods. Variant demand is presented in Fig. 1. The move suppression term would definitely affect control performance leading to a more sluggish dynamic response. In this part, centralized MHC method is applied to the supply chain network with a variant customer demand that is seeing in figures 2 and 3.

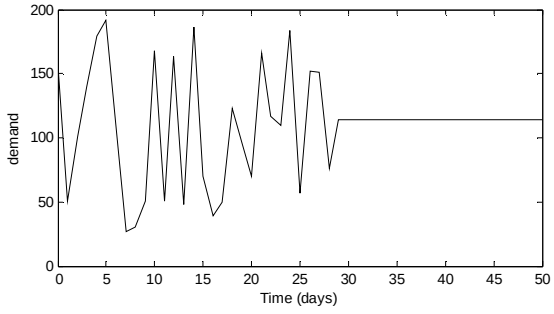

Fig. 1 Demand

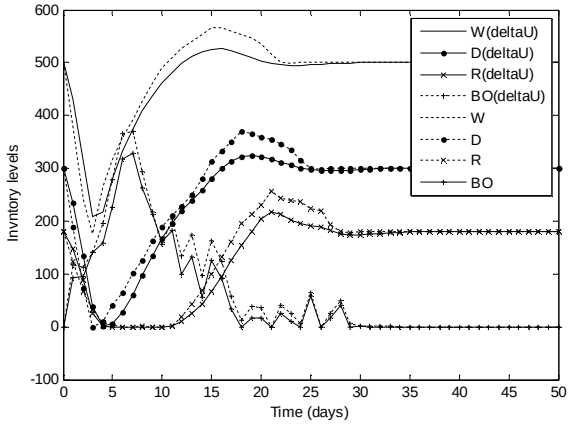

Fig. 2 MHC outputs of supply chain management system:deltaU curves is with move suppression effect

CONCLUSION

The large majority of successful MHC applications address the case of multivariable control in the presence of constraints, motivating its extensive distribution for applications where traditional control usually comes close to its limits. The success of MHC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite horizon strategy allows the explicit handling of process and operational constraints by the MHC. Typically, MHC is implemented in a centralized fashion. In this paper, a centralized moving horizon controller applying to a supply chain management system consist of one plant (supplier), two distribution centers and three retailers. Also a move suppression term add to cost function, that increase system robustness toward changes on demands. Through illustrative simulations with variation of demand, it is demonstrated that move suppression effect decreases maximum changes of customer demands.

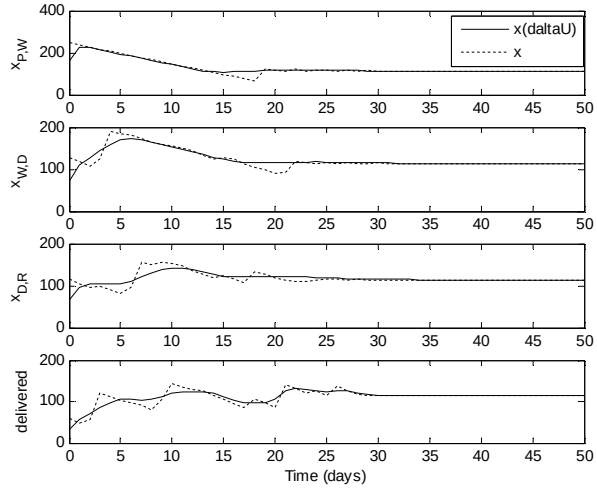

Fig. 3 MHC inputs of supply chain management system:deltaU curves is with move suppression effect

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